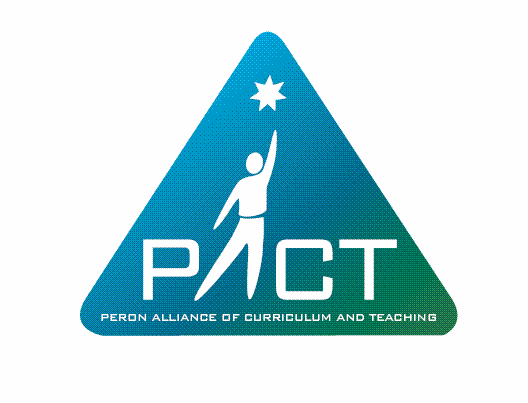
Name\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: *\_\_*

50

= %

**METHODS MAT 11**

**Test 5 - 2015**

**Topics: Rates of Change ,Differentiation & Application of Differentiation**

**Total Time:**  *55 minutes*  **Weighting:** *6% of the year.*

*This test comprises of* ***TWO sections****. The* ***first section*** *is* ***calculator free*** *where no calculators of any kind are to be used. The* ***second section*** *is* ***calculator assumed*** *where the CAS calculator may be used. All questions must be answered in both sections.* ***Answers should be rounded to 2 decimal places unless specified****. All working should be shown in the space provided. Solutions without working may not be awarded full marks. Please take the marks for each question into account when answering the question.*

**SECTION 1: CALCULATOR FREE**

**Time:** 20 minutes**Equipment Allowed:** Formula sheet

**Marks for Section 1:** *22marks*

1. (3 marks:1,2 )

Find: (a) (b)

13 🗸 🗸

= -4 🗸

2. (5 marks)

From first principles, find the gradient of the function f(x) = 3x2 at the point (-1,3).

f’(x) =

=

= 🗸

= 🗸 🗸

= 6x 🗸

|  |  |  |  |
| --- | --- | --- | --- |
| 3. | [6 marks: 3, 2,1] | | |
|  | i) For the points of the graph above determine if they are: Local Minima , local maxima, x intercept, y intercept, horizontal point of inflection. | | |
| **A** | x intercept |  |  |
| **B** | y intercept, local maxima |  |  |
| **C** | x intercept |  |  |
| **D** | Local Minima |  |  |
| **E** | x intercept |  |  |
| **F** | horizontal point of inflection |  |  |

🗸 🗸 🗸 (-1/2 for each mistake)

ii) At which points on the curve is the gradient of the tangent to curve equal to zero?

B,D, F 🗸 🗸 (-1/2 for each mistake)

iii) Between which points is the gradient of the curve negative?

B and D 🗸

4. (5 marks: 2,3)

Differentiate with respect to x:

(a) y = x5 – x3 +7x +2 (b) y=

= 5x4 – 3x2 + 7 🗸🗸 y = +

y= 3x3 + x2 🗸🗸

= 9x2 + 2x 🗸

5. (3 marks)

If f(x) = 3x2 – 4x, find the value of a given that f**ꞌ**(a)=5

f’(x) = 6x -4 🗸

f’(a) = 6a -4

6a-4 = 5 🗸

a= 3/2 🗸

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**SECTION 2: CALCULATOR ASSUMED**

**Time:** *30 minutes* **Equipment Allowed:** Formula sheet, *1 page of notes (A4), CAS and scientific calculators*

**Marks for Section 2:** *28 marks*

7. (2 marks)

Find =

= 2 🗸🗸

8. (2 marks)

Find the average rate of change for the expression f(x)= x2 -6x +10 from x = 4 to x=4.001

f(4)= 2 f(4.001)= 2.001

Average rate of change = = 2 🗸🗸

9. (5 marks: 1,1,1,2)

A colony of bacteria is increasing in such a way that the number of bacteria present

after t hours is given by N where N = 120 + 500t + 10t3. Find

(a) the number of bacteria present initially 120 🗸

(b) the number of bacteria present when t= 5 3870 🗸

(c) the average rate of increase, in bacteria/hour, in the first 5 hours 750 bacteria/hr 🗸

(d) the rate the colony is increasing, in bacteria/hour, when t= 10 3500 bacteria /hr 🗸🗸

10.(3 marks)

Find the gradient of the curve y= 2x3 +5 at the point where y=3

At y = 3, x = -1🗸

dy/dx = 6x2 🗸

gradient = 6 🗸

11. (3 marks)

Determine, using calculus, the coordinates on the curve y= 2x3 -3x2 +4 where

the gradient is 0.

dy/dx = 6x2 – 6x 🗸

When 6x2 -6x = 0, x = 0 or x = 1

Required coordinates = ( 0,4) and ( 1,3) 🗸🗸

12. (4 marks)

|  |
| --- |
| Use your CAS calculator to sketch the graph of . |
| Hence, find: |
| (a)the y-intercept (0,-5) 🗸 |
|  |
| (b) the x-intercept(s) (4.1, 0) 🗸 |
|  |
| (c) the turning point(s), stating whether they are a maximum or minimum |

(3,-5) Min 🗸 (1,-1) Max 🗸

13. (9 marks:3,3,3)

A piece of wire, 300cm long, is used to make the 12 edges of the frame of a rectangular box. The length *(L)* of the rectangular frame is 3 times the width *(x)* of the frame.

1. Show that the height of the rectangular box is given by: .

4L + 4*x* + 4h = 300 🗸

L + *x* + h = 75

Now L = 3*x* 🗸

3*x* + *x* + h = 75, h = 75 -4*x* (shown) 🗸

1. Show that the volume, *V*, of the box is given by .

V =L × *x* ×h 🗸

V = 3x × x × ( 75 – 4x) 🗸

V = 225 x2 – 12 x3 (shown) 🗸

1. Find the width of the frame that will maximise the volume of the box and find this maximum volume.

For max volume, dv/dx = 0

dv/dx = 450x – 36x2 🗸

450x -36x2 = 0 🗸

X= 12.5

Width of frame = 12.5 cm 🗸